RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER TAKE-HOME TEST/ASSIGNMENT, JULY 2020

FIRST YEAR (ARREAR)

Date : 10/07/2020 **MATH FOR INDUSTRIAL CHEMISTRY (General)**

Time : 11am - 1pm **Paper : II** Full Marks : 40

Instructions to the Candidates

- Write your Name, College Roll no, Subject and Paper Number on the top of the Answer Script and on the text body of the mail.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be admissible.
- Each paper/group must be answered in a single booklet.
- All the pages of your answer scripts must be numbered serially by hand.
- In the last page of your answer-scripts, please mention the total number of pages written so that we can verify it with that of the scanned copy of the scripts sent by you.
- For an easy scanning of the answer scripts and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the test, you should scan the entire answer scripts by using Clear Scan : Indy Mobile App /other Scanner devices and make a file in your own name and send or share them as a PDF file to [redacted]

Group - A (Vector Algebra)

Answer any two questions from this group. **[4 x 2 = 8 marks]**

1. If $\vec{\alpha} = \hat{i} + \hat{j} - 6\hat{k}$, $\vec{\beta} = \hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{\gamma} = 2\hat{i} - 5\hat{j} + 3\hat{k}$, find $(\vec{\alpha} \times \vec{\beta}) \times \vec{\gamma}$. [4]
2. Find the vector equation of the plane through the point $(2, 3, -1)$ and perpendicular to the vector $3\hat{i} - 4\hat{j} + 7\hat{k}$. [4]
3. Find the vector equation of a straight line passing through the points $\hat{i} - 2\hat{j} + \hat{k}$ and $3\hat{k} - 2\hat{j}$. [4]

Group - B (Differential Calculus)

Answer any two questions from this group. **[6.5 x 2 = 13]**

4. (a) Check the convergence of the series

$$\frac{1}{3 \cdot 7} + \frac{1}{4 \cdot 9} + \frac{1}{5 \cdot 11} + \frac{1}{6 \cdot 13} + \ldots$$

where $\cdot$ represents product in the above expressions. [4]

(b) Justify with proper reasoning whether the following statement is true or false :
"Given two sequences $\{a_n\}$ and $\{b_n\}$, if the product sequence $\{a_n b_n\}$ converges, then at least one of $\{a_n\}$ or $\{b_n\}$ is convergent." [2.5]

5. (a) Check whether the sequence given by $u_1 = \sqrt{3}$, $u_{n+1} = \sqrt{3u_n}, \forall n \geq 1$, converges or not? Find its limit, if it exists. [3.5]

   (b) Check the convergence of the series [3]

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ...$$

6. Find maxima and minima of the function

$$f(x, y, z) = (x - a)^2 + (y - b)^2 + z^2$$

subject to the constraint $ax + by + z = 1$. [6.5]

Group - C (Integral Calculus I)

Answer any two questions from this group. **[3 x 2 = 6 marks]**

7. Evaluate

$$\int \frac{18 \sin x + \cos x + 10}{3 \sin x - 4 \cos x + 5} dx$$

. [3]

8. If

$$u_n = \int \frac{dx}{(x^2 + 1)^n},$$

show that [3]

$$(2n - 2)u_n - (2n - 3)u_{n-1} = \frac{x}{(x^2 + 1)^{n-1}}$$

.

9. Evaluate

$$\int_0^1 (ax^2 + bx + c)dx$$

by the method of summation. [3]

Group - D (Probability Theory)

Answer all the questions from this group. **[3 + 5 + 5 = 13 marks]**

10. Find the probability that a leap year chosen at random will contain 53 Mondays. [3]

11. The download time of a resource web page is normally distributed with a mean of 6.5 seconds and a standard deviation of 2.3 seconds. What is the probability that the download time will be between 4 and 10 seconds? [5]

12. Let the random variable X have the following pdf

$$f(x) = \begin{cases} cx(2 - x), & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find c and calculate the probability $P(0.5 \leq X \leq 1)$. [5]

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